Simple theory for the Levitron®

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The Levitron® is a popular toy that provides a unique and interesting demonstration of levitation using permanent magnets. It consists of a small, spinning magnetic top and a magnetic base plate. Stable levitation is possible because of a unique coupling of the magnetic forces and torques with the gyroscopic action of the top. A simple theory of its operation, using a general axisymmetric form for the magnetic field of the base, is based on the dipole force model. With this model, the stable behavior of the spinning, levitated top may be investigated by testing various assumptions for its orientation. Stability is not possible if the top remains rigidly parallel to the axis of the base. On the other hand, if one assumes that the top remains aligned parallel to the local magnetic field during radial excursions, then stability is possible. This simple model, combined with measurements of the magnetic field along the axis, permits fairly accurate prediction of the upper and lower limits of the locus of stable equilibria. © *1997 American Institute of Physics.* [S0021-8979(97)05814-3]

I. INTRODUCTION

The Levitron® is a very interesting toy that provides a remarkable demonstration of stable magnetic levitation.[1] It consists of a small top (a nonmagnetic spindle inserted in a flat, toroidally shaped, permanent magnet), a square plastic base (with ceramic magnet imbedded within it), and a plastic lifter plate (used to raise the spinning top for insertion into levitation). The base, magnetized perpendicular to the plane surface, is square and has a large, circular hole or unmagnetized region at its center. See Fig. 1, which shows the device in half-section, along with some magnetic field lines. Through gyroscopic action, the spinning top is maintained in nearly vertical alignment so that the strong dipole–dipole repulsion force can suspend it against gravity without being flipped over. The device is readily available from toy stores and mail-order establishments, and has been written up widely.[2] There exist several web sites devoted specifically to the Levitron®, one containing a short video that can be downloaded and others describing its physics.[3]

Virtually any claim of levitation is guaranteed to gain attention. For example, there is an uncannily believable account written by an 11th century Arab scholar of an ancient Hindu temple at Somanantha in India containing a sacred shrine suspended by an array of lodestone magnets.[4] Some contemporary scholars dismiss this claim,[5] but, whatever the truth about Somanantha, it is clear that controversy about levitation has a long history. Interest in levitation today ranges from futuristic MAGLEV trains to the Sunday funnies. While not immune to curiosity about it, engineers and scientists by training harbor a certain skepticism regarding levitation. This skepticism may be predicated upon awareness of Earnshaw's theorem, a well-known property of curl- and divergence-free fields that precludes the existence of local, detached scalar potential maxima or minima.[6] This theorem teaches that it is impossible to levitate a charged particle *statically* in space. There is a tendency to extend the implications of this rule inappropriately to other types of levitation, both electric and magnetic. Such thinking is clearly erroneous; charged particles can be levitated dynamically, and feedback-controlled systems are used to levitate magnetic bearings. Furthermore, passive and feedback-controlled levitation of uncharged particles, droplets, and bubbles suspended in dielectric fluids is well-documented.[7]

The objectives of this paper are, first, to provide a detailed phenomenological description of the toy and, second, to present an intuitive model that successfully predicts certain behavior of the Levitron® and provides important clues about what governs its complex dynamics. Our model is consistent with the works of Berry[8] and Simon *et al.*[9] In the present paper, emphasis is placed on the magnetic field and the requirements imposed upon it for successful levitation. On-axis magnetic field measurements are used to predict the locus of stable levitation.

II. PHENOMENOLOGY

The instructions for the Levitron® advise that consistently getting the top to spin smoothly without wobble on the lifter plate is challenging and requires considerable patience. The difficulty stems from the very strong magnetic torque that must be overcome to obtain clean, wobble-free rotation at the center of the base plate. Second, the instructions specify that the lifter plate, with the top spinning on it, be raised very slowly. Third, the weight of the top must be finely adjusted so that it just barely lifts off the plate when it reaches the lower limit of the stable region. That insertion into stable levitation is so difficult is an indication that the locus of stability is small and that too much translational energy imparted to the top will prevent its being trapped

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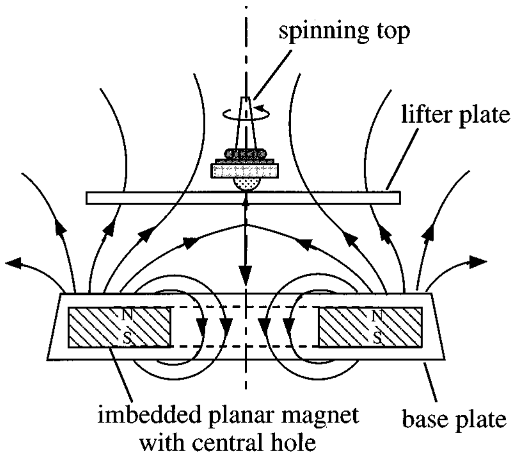

FIG. 1. The Levitron® toy, shown here in sectional view, consists of a top with toroidal magnet, a lifter plate, and a base plate with an imbedded, square ceramic magnet. Some field lines of the base plate magnet are shown. Note the zero and positive maximum of $B_z=\mu_0 H_z$ along the axis.

successfully. The simple model presented below, when combined with some magnetic field measurements and experiments, confirms that the stable region is indeed small.

If the magnetic field axis of the base plate is well-aligned with the vertical, the weight of the top properly adjusted, and the lifter plate raised slowly, the top can remain levitated for up to 5 min in air (much longer in vacuum and indefinitely when driven[9]). As air friction slows the top, a rapid wobbling motion eventually becomes visible. This wobble grows until the top falls, usually straight downward. The vertical elevation of the top's center of mass does not change perceptibly as the rotation slows until the angle of the wobble (with respect to vertical) has increased to between ~3° and ~5° (or 0.05–0.09 rad). Measurement of the rotation speed at which the top falls, achieved using a General Radio Strobotac™, yielded reasonably consistent results in the range of 1100–1200 rpm (18–20 Hz), with the variation dependent on the weight of the top. In our experiments, we made only minimal efforts to damp out initial wobble.

The importance of vertical alignment of the magnetic axis is evident, both when the top is initially levitated and later as it slows down. Sometimes, levitation can be prolonged by using the leveling wedges to reduce visible precessional motion, a tricky adjustment that apparently improves the alignment of the magnetic base normal to the gravitational field.

Proper weighting of the top is very critical to successful levitation. If too heavy or too light, no equilibrium position exists at all. Furthermore, trapping the top when its weight is at the light end of the range is difficult because the top jumps off the lifter plate too energetically for capture. Adding or removing one small black ''O'' ring (weighing ~0.06 g) from the top changes the axial position by approximately 1 mm; the full realizable length of the stable locus (~4 mm) is covered by a ~1% change in the mass. If one levitates the top when its mass is near the lower limit, persistent, apparently random motion is observed. This motion, consisting of coupled side-to-side and up-and-down oscillations, is imparted to the top as it is released into the equilibrium. The apparent period of this oscillation is of the order of 1 s. Sometimes, after levitating ~10 s or even more, the top will make a large radial excursion and slip out of the trap. Radial excursions of the levitated top from the central axis are always accompanied by precessional motion and nutation. Observed from above, the upper end of the top prescribes small cardioids superimposed on the circular precessional motion. The nutation suggests enforcement of an effective, time-averaged alignment of the magnetic moment of the top with the magnetic field lines, which diverge from the axis.

The earth's magnetic field is far too small to influence the device; however, rather small magnets or magnetizable objects placed near the spinning top or the base can perturb the field sufficiently to destroy the equilibrium. The instructions warn of the need for almost constant readjustment of the weight of the top, and this effect has been ascribed to small temperature changes that influence the permanent magnets;[8,9] however, another factor might be the sensitivity of the magnetic field to the steel in table tops or other furniture located near the toy. For example, if the magnetic base is placed atop a 6.35-mm-thick steel plate, both the magnitude and shape of the field are very significantly altered. The equilibrium position of the top is shifted downward approximately 3 mm and the weight of the top must be increased >25%.

III. THEORY

The curl- and divergence-free magnetic field of the square base is nearly axisymmetric near the axis. Taking $z=0$ as the equilibrium position, the axial (H_z) and radial (H_ρ) components are:[10]

$$H_z=H_0[1+\alpha_1 z'+\alpha_2(z'^2-\rho^2/2)+\cdots], \tag{1a}$$

$$H_\rho=H_0[-\alpha_1\rho/2-\alpha_2 z'\rho+\cdots]. \tag{1b}$$

Here, z' and ρ represent, respectively, small axial and radial excursions of the particle from equilibrium and H_0 is the axial field strength at $z=0$, $\rho=0$. The coefficients α_1, $\alpha_2,\ldots$, are simply related to the axial derivatives of H_z,

$$\alpha_n(z_0)=\frac{1}{n!}\left[\frac{1}{H_z}\frac{\partial^n H_z}{\partial z^n}\right]_{\rho=0,z=z_0}. \tag{2}$$

This definition for the coefficients may be used readily with numerical field data, either measured or computed. With no loss of generality, we assume $H_0>0$.

A. Equilibrium

Ignoring higher order multipoles, the force exerted by the magnetic field $\overline{H}$ on a dipole of moment $\overline{m}$ in air or vacuum depends on the directed gradient of the field:[7]

$$\overline{f_m}=\mu_0\overline{m}\cdot\nabla\overline{H}, \tag{3}$$

where $\mu_0=4\pi 10^{-7}$ H/m is the permeability of free space. For equilibrium to exist

$$\mu_0\overline{m}\cdot\nabla\overline{H}-Mg\hat{z}=0, \tag{4}$$

where M is the mass of the top and $g=9.81$ m/s^2 is the terrestrial gravitational acceleration. Employing the field expansion, Eq. (1a), and assuming that $\overline{m}=m\hat{z}$ (with the sign of m unspecified)

$$\mu_0 m \alpha_1 H_0 = Mg. \tag{5}$$

Because $Mg>0$, then $m\alpha_1>0$ is required for equilibrium.

B. Stability

Positional stability of the equilibrium defined by Eq. (5) imposes additional restrictions on α_1 and α_2. We can establish these conditions by a perturbation analysis upon $\overline{f_m}(\rho,z)$. To proceed, some assumption must be made about the vector moment $\overline{m}$ of the top. Because the top is a permanent magnet, it is justified to assume that $|m|$ is constant. On the other hand, the orientation of the spinning top is controlled by the interplay of magnetic and gyroscopic torques. Here, however, we limit consideration to two limiting cases, both of which assume orientational stability as a given.

The first model assumes that the top is spinning so rapidly that, irrespective of radial or axial excursions of the top from its equilibrium position, gyroscopic action maintains the magnetic moment vector in perfect vertical alignment, that is, $\overline{m}\|\hat{z}$. Under this condition, Eq. (3) for the dipole force on the top becomes

$$\overline{f_m} = \mu_0 m \frac{\partial}{\partial z} \overline{H}. \tag{6}$$

Using the field expansion, Eqs. (1a) and (1b), and subtracting out the equilibrium condition, Eq. (5), we obtain for the perturbation force vector

$$\overline{f'_m} = \mu_0 m H_0 [2\alpha_2 z' \hat{z} - \alpha_2 \rho \hat{\rho}]. \tag{7}$$

If $m\alpha_2<0$, the top will be stable to axial displacements but unstable to radial, while if $m\alpha_2>0$, the top will be stable to radial but not to axial displacements. Thus, the $\overline{m}\|\hat{z}$ constraint cannot lead to stability. Invoking Earnshaw's theorem with essentially the same assumption about alignment, Berry arrived at a similar conclusion.[8] The fact that a rigidly oriented top is always unstable means that, in addition to the obvious lower limit, there exists an *upper* limit to the rotation rate of the top for stable levitation.[8,9]

A second candidate for a model, qualitatively grounded in observation, specifies that the top remain parallel to the magnetic field H of the base plate for all excursions from equilibrium,

$$\overline{m} = m \frac{\overline{H}}{H}, \tag{8}$$

where $H=|\overline{H}|$. Combining Eqs. (3) and (8), the expression for the force on the top becomes

$$\overline{f_m} = \frac{\mu_0 m}{2} \frac{\nabla H^2}{H}. \tag{9}$$

To proceed, some additional field expansions based on Eqs. (1a) and (1b) are used[11]

$$\frac{\partial H^2}{\partial z} = H_0^2[2\alpha_1 + 2(\alpha_1^2 + 2\alpha_2)z' + \cdots], \tag{10a}$$

$$\frac{\partial H^2}{\partial \rho} = H_0^2\left[2\left(\frac{1}{4}\alpha_1^2 - 2\alpha_2\right)\rho + \cdots\right], \tag{10b}$$

$$H = |\overline{H}| = H_0[1 + \alpha_1 z' + \cdots]. \tag{10c}$$

Using these expansions with Eq. (9), and subtracting out the equilibrium condition, the perturbation force components, correct to linear terms, are

$$f'_{m,z} \cong 2\mu_0 m H_0 \alpha_2 z', \tag{11a}$$

$$f'_{m,\rho} \cong \mu_0 m H_0 \left(\frac{1}{4}\alpha_1^2 - \alpha_2\right)\rho. \tag{11b}$$

For both the axial and radial perturbation forces to be restoring in nature,

$$m\alpha_2 < 0, \tag{12a}$$

$$m(\alpha_1^2 - 4\alpha_2) < 0. \tag{12b}$$

Simultaneously satisfying these inequalities and the equilibrium condition $m\alpha_1>0$ is possible only if (i) $m<0$ and (ii) three conditions on the coefficients are met,

$$\alpha_1 < 0, \tag{13a}$$

$$\alpha_2 > 0, \tag{13b}$$

$$\alpha_1^2 > 4\alpha_2. \tag{13c}$$

There is no incompatibility among these inequalities and therefore, because α_1 and α_2 are independent, stability is possible with a properly designed axisymmetric permanent magnet. Berry invokes a time-average approach for which, in our notation $\overline{m}\cdot\mu_0\overline{H}$ becomes invariant.[8] His hypothesis differs from Eq. (8) by the cosine of a small angle. In either case, the consequences of Earnshaw's theorem are avoided by the constraint that radial excursions of the top are always accompanied by sufficient tilt to preserve the orientation of $\overline{m}$ with the local $\overline{H}$.

C. The magnetic field of the base plate

In the Levitron®, the magnetic field of the base is provided by a permanently magnetized square plate with a centered, demagnetized region or ''hole.'' For convenience, we use a simple ring dipole model. Any quantitative differences between this model, or the circular disk employed by Berry,[8] and the actual base magnet are certainly overwhelmed by larger errors due to the finite size of the magnet in the top. The ring dipole can be thought of as a line dipole bent into a circle and with its magnetic axis oriented parallel to $\hat{z}$. By simple adaptation of an analogous electrostatics problem,[12] we have:

$$H_z = \frac{m_{\text{ring}}}{4\pi R^3}\left\{\frac{2(Z/R)^2 - 1}{[(Z/R)^2+1]^{5/2}}\right\}, \tag{14}$$

where m_{ring} and R are, respectively, the dipole moment and radius of the ring, and Z measures the vertical distance from the plane of the ring. Just as suggested by Fig. 1, the field is

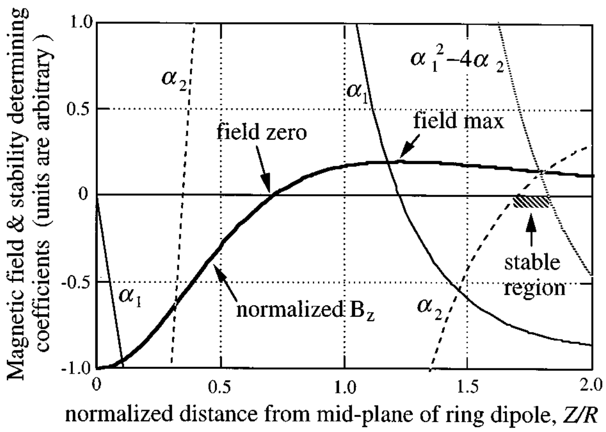

FIG. 2. Theoretical plot from Eq. (14) of the magnetic field intensity $B_z = \mu_0 H_z$ on the z axis for the ring dipole with locus of stable equilibria also shown.

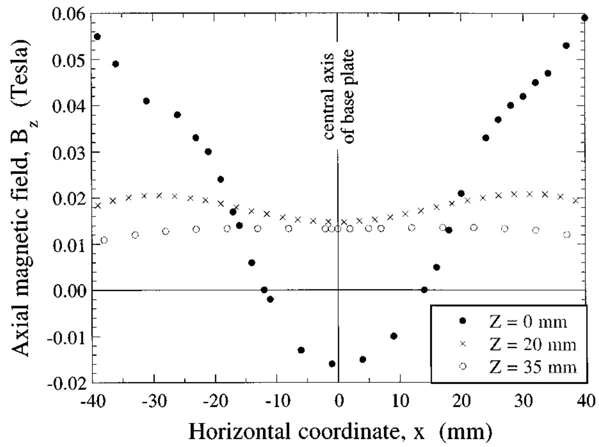

FIG. 3. Profile of axial magnetic field magnitude B_z vs x obtained at three elevations above the Levitron® base plate: $Z'=0$, 20, and 35 mm. The $Z'=0$ data clearly reveal the presence of the large, demagnetized volume in the center of the square base magnet. Contrast this with the virtual uniformity of B_z at $Z'=35$ mm.

negative near $Z=0$, goes through zero at $Z=R/\sqrt{2}$, reaches a positive maximum at $Z=\sqrt{3/2}R$, and asymptotically approaches zero as $Z\rightarrow\infty$. The locus of stable equilibria for the ring dipole, defined by Eqs. (13b) and (13c), lies on the axis and is confined to $1.69R>Z>1.82R$. See Fig. 2.

The prediction of a stable locus for an axisymmetric geometry would seem to contradict one claim made in the patent by Hones and Hones that levitation cannot be achieved with a base magnet having a periphery of circular shape.[1] The present analysis, as well as that of Berry[8] and Simon *et al.*,[9] offers clear evidence that axisymmetric magnetic fields should work fine. In fact, success in predicting stable levitation for axisymmetric fields suggests that there is no fundamental distinction between the Levitron® and the much earlier scheme of Harrigan.[13] Given the practical potentialities of this type of magnetic levitation, it might be worthwhile to search for an optimized base magnet geometry by exploiting numerical field calculation techniques. Finally, the model shows that it is unnecessary to invoke eddy current effects to explain the radial stability of the Levitron®.[14]

IV. EXPERIMENT

Our quantitative investigation of the Levitron® toy, though limited to measurements of the magnetic field and the equilibrium position versus the mass of the top, provides the means to test the model. The magnetic field was measured using a transverse Hall-effect gaussmeter probe mounted on a computer-controlled xyz positioner having mechanical resolution better than ± 0.01 mm as long as backlash is avoided. All magnetic field readings were accurate to $\pm 2\%$. Figure 3 shows $B_z=\mu_0 H_z$ data for horizontal traverses made with the probe aligned horizontally at three different elevations: $Z'=0$, 20, and 35 mm. Note that Z' was measured from the surface of the magnetic base, rather than from the midplane as specified for the idealized ring dipole. These data, plus those from similar traversals in the y direction, reveal a reversal of the field close to the plate near the centerline due to the fairly large ''hole'' in the square permanent magnet. Figure 4 shows B_z measured on the axis ($\rho=0$) as a function of Z'. Consistent with Fig. 1 and Eq. (14), the axial magnetic field is negative close to the base plate, reverses sign, reaches a positive maximum, and then asymptotically approaches zero for increasing Z'.

Exploiting a least-squares curve fitting routine to obtain a sixth-order polynomial expression for $B_z(Z')$, we computed the coefficients α_1 and α_2 using Eq. (2) and then employed Eqs. (13b) and (13c) to determine the locus of stability. This locus, shown in Fig. 5(a) and extending from ~40 to ~47 mm, was compared to the experimentally observed locus by observing the position of the top as the mass was adjusted. These experiments were performed rapidly to minimize any equilibrium ''drift'' due to temperature changes, etc. With mass $M=20$ g, the top levitated at 42 (± 0.5) mm; then, one ''O'' ring (~0.06 g) was added at a time until levitation was no longer achievable. The lower limit at $Z'=39.5$ (± 0.5) mm was reached for $M=20.2$ g. Great difficulty is encountered levitating the top when its mass is near the low end of the acceptable range, so the observed value of

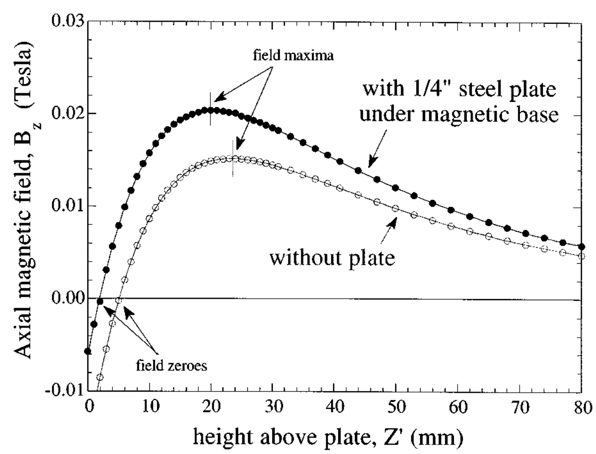

FIG. 4. Axial magnetic field B_z vs elevation Z' above the Levitron® base plate alone and with a 15 cm square, 6.35-mm-thick steel plate inserted below the base plate. The continuous curves are sixth-order polynomials fitted to the data.

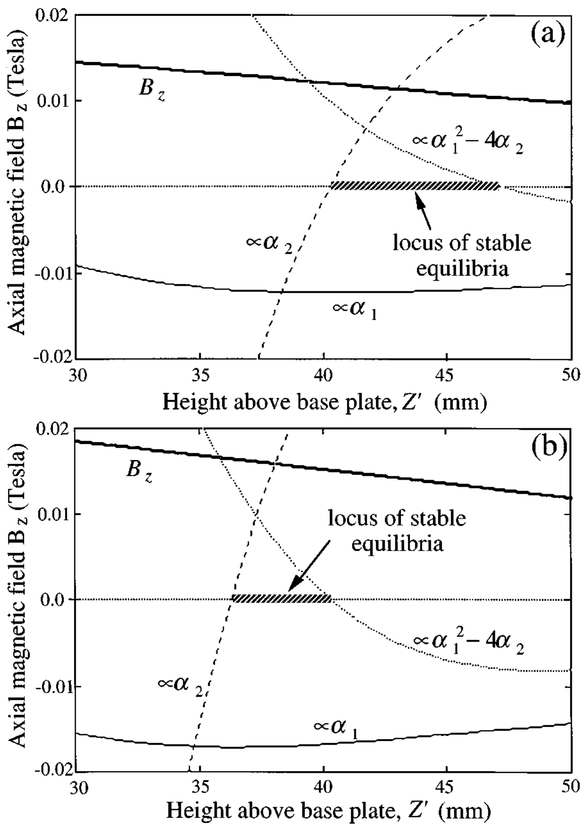

FIG. 5. Plot of $B_z(Z)$ on the axis and near the locus of stable equilibria for the permanent magnet base of the Levitron®. All curves are based on the sixth-order fitted polynomials for $B_z(Z')$ shown in Fig. 4. In considering stability, only the signs of α_1, α_2, and $(\alpha_1^2-4\alpha_2)$ are important, so these quantities are plotted with arbitrary units: (a) as is, without the steel plate; (b) with 15 cm square, 6.35-mm-thick steel plate.

42 mm is not the true upper limit of the stable locus. The calculated and measured results are summarized in Table I.

A further test of the equilibrium and stability theory was conducted by placing the magnetic base plate upon a 15 cm square plate of 6.35-mm-thick cold-rolled steel. Figure 4 shows the effect of the plate on the magnetic field B_z. We again fitted the data to a sixth-order polynomial curve and then computed the coefficients α_1 and α_2 using Eq. (2). As shown in Fig. 5(b), the predicted stable locus, ranging from ~36.5 to ~40.5 mm, is closer to the plate. In addition, a comparison of Figs. 5(a) and 5(b) shows that α_1 increases by about 40% with the steel plate in place, indicating that the weight of the top must be increased by the same percentage to be levitated. As shown in Table I, these predictions correlate rather well to experimental measurements. To achieve levitation, the top's mass had to be increased by almost 30% and the observed locus of levitation was from ~36.5 (±0.5) to ~39.5 (±0.5) mm for mass values ranging from 25.5 to 25.2 g, respectively.

V. CONCLUSION

In this paper, we have employed a simple dipole interaction model and a Taylor series expansion for the magnetic field to predict certain features of the equilibrium and stability of the Levitron®. We use simple means to show that the top cannot be stably levitated when rigidly aligned with the vertical axis, and that, therefore, the top must be free to tilt as it moves off-axis. Because gyroscopic resistance to tilt is directly related to rotation speed, we conclude that there must exist an upper as well as a lower limit to the rotation rate for levitation. An assumption that the top remains parallel to the local magnetic field, Eq. (8), for all excursions from the axis, leads to the prediction of stable levitation. This model, when combined with numerical data for the variation of the magnetic field along the axis, gives a reasonably accurate prediction of this locus. In addition, two key experimental observations—that the locus is small (~4 mm in length) and that the mass of the top must be within a narrow range (~1%)—are clearly reflected in the model's predictions.

It is obvious that the dynamics of the Levitron® are far more complex than intimated by Eq. (8). While the top does tend to align itself with the magnetic field as it moves off-axis, gyroscopic precession and nutation are clearly evident. This gyroscopic action is further complicated by strongly coupled axial and radial motions. While our simple model has ignored these motions, the more sophisticated theories of Berry[8] and Simon *et al.*[9] must be invoked to investigate more fully the dynamics of this fascinating toy.

ACKNOWLEDGMENTS

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TABLE I. Summary of calculated and measured locations of zeroes and maxima of axial magnetic field B_z on the axis, plus the limits of stable equilibria of the Levitron® base plate as supplied and with 15 cm square, 6.35-mm-thick cold-rolled steel plate.

	Location of zero of B_z	Location of max. of B_z	Lower limit of stable locus	Upper limit of stable locus
Without plate	5 mm	23.5 mm	Computed: 40 mm Measured: 39.5 mm	Computed: 47 mm Measured: 42 mm[a]
With 6.35-mm-thick steel plate	2 mm	20 mm	Computed: 36.5 mm Measured: 36.5 mm	Computed: 40.5 mm Measured: 39.5 mm[a]

[a]Experimental determination of the upper end of the locus of stable equilibria is difficult due to the extreme sensitivity of the equilibrium to any initial motion imparted to the top when it is first inserted into the trap. The values provided here are certainly below the true upper limit.

tley, who made available the xyz computer-controlled, positioning apparatus and then assisted in making the magnetic field measurements. We also acknowledge M. D. Simon, who freely shared his extensive knowledge of spin stabilized magnetic levitation.

[1] E. W. Hones and W. G. Hones, U. S. Patent No. 5,404,062 (4 April 1995).
[2] R. Edge, Phys. Teach. **33**, 252 (1995); **34**, 329 (1995); C. Ucke and H.-J. Schlichting, Phys. Unseren Zeit. **26**, 217 (1995).
[3] http://www.edoc.com/dan/levitron.html; http://popularmechanics.com/popmech/sci/tech/U086G.html; http://www.lauralee.com/physics.htm
[4] R. Thapar, *A History of India* (Pelican, Aylesbury, Bucks, UK, 1982), Vol. 1, pp. 233–234.
[5] K. M. Munshi, *Somanatha—The Eternal Shrine* (Bharatiya Vidya Bhavan, Bombay, 1976), p. 141.
[6] J. C. Maxwell, *Treatise on Electricity and Magnetism*, 3rd ed. (Dover, New York, 1954), Art. 116.
[7] T. B. Jones, *Electromechanics of Particles* (Cambridge University Press, New York, 1995).
[8] M. V. Berry, Proc. R. Soc. London, Ser. A **452**, 1207 (1996).
[9] M. D. Simon, L. O. Heflinger, and S. L. Ridgway, Am. J. Phys. **65**, 286 (1997).
[10] R. E. Holmes, J. Appl. Phys. **49**, 3102 (1978).
[11] T. B. Jones, J. Electrostat. **11**, 85 (1981).
[12] E. Weber, *Electromagnetic Theory* (Dover, New York, 1965), pp. 125–127.
[13] R. M. Harrigan, U.S. Patent No. 4,382,245 (3 May 1983).
[14] C. Murakami, *Proceedings of the Eighth Symposium on Electromagnetics and Dynamics* (Institute of Mechanical Engineers, Tokyo, 1996), p. 461.